AE353: Midterm

October 7, 2024

You are allowed to use any online or offline notes at your disposal, **but no communication and computational tools**. You must **write a complete solution, showing the process that you took to reach your answer**. **"Bonus" problem is not necessarily the hardest.**

Problem 1. (25 points)

Consider a controlled dynamical system with state $q \in \mathbb{R}$ and input $p \in \mathbb{R}$ given by the equation

$$\dddot{q} = 2\dot{q}\ddot{q}p^2 - \cos(p^2).$$

By adding "dummy states", convert it into a first-order dynamical system described by equations $\dot{w} = f(w, p)$, where w is the extended system state. *Write the definition of w, and carefully write every component of f in terms of w and p.*

Problem 2. (25 points)

Consider a nonlinear control system with state $w \in \mathbb{R}^2$ and input $p \in \mathbb{R}$ given by

$$\begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} = \begin{pmatrix} e^{w_2} + w_1 p^2 - 1 \\ w_1 w_2 \end{pmatrix},$$

with output $z \in \mathbb{R}$ given by $z = e^{w_2}$. Linearize this system around an equilibrium (w_e, p_e) such that $p_e = 1$. Express the linearized system in the form $\dot{x} = Ax + Bu$, $y = Cx + Du$, and make sure to write down the relationships between x and w, u and p, and y and z.

Problem 3. (25 points)

Consider the linear control system

$$\dot{x}(t) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(t),$$

with $x(0) = \begin{pmatrix} 2 & 3 \end{pmatrix}^T$. Let

$$u(t) = \begin{pmatrix} 0 & -1 \end{pmatrix} x(t)$$

for all t. Determine, *without use of computational tools*, $x(5)$.

Problem 4. (25 points)

Consider the linear control system

$$\dot{x}(t) = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(t).$$

(a) Determine whether this system is controllable.

(b) Determine whether this system is stabilizable.

(c) Determine whether it is possible to design a control signal which ensures $x_2(t) \to 1$.

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Bonus Problem. (25 points)

Consider a controlled dynamical system with state $z \in \mathbb{R}$ and input $\tau \in \mathbb{R}$ described by

$$\dot{z} = 7z^{12} + 5z + \tau^2.$$

Let $w = z^3$. Describe the dynamics of w, i.e., find a function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $\dot{w} = f(w, \tau)$.